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Contextual affordances in context-aware autonomous systems

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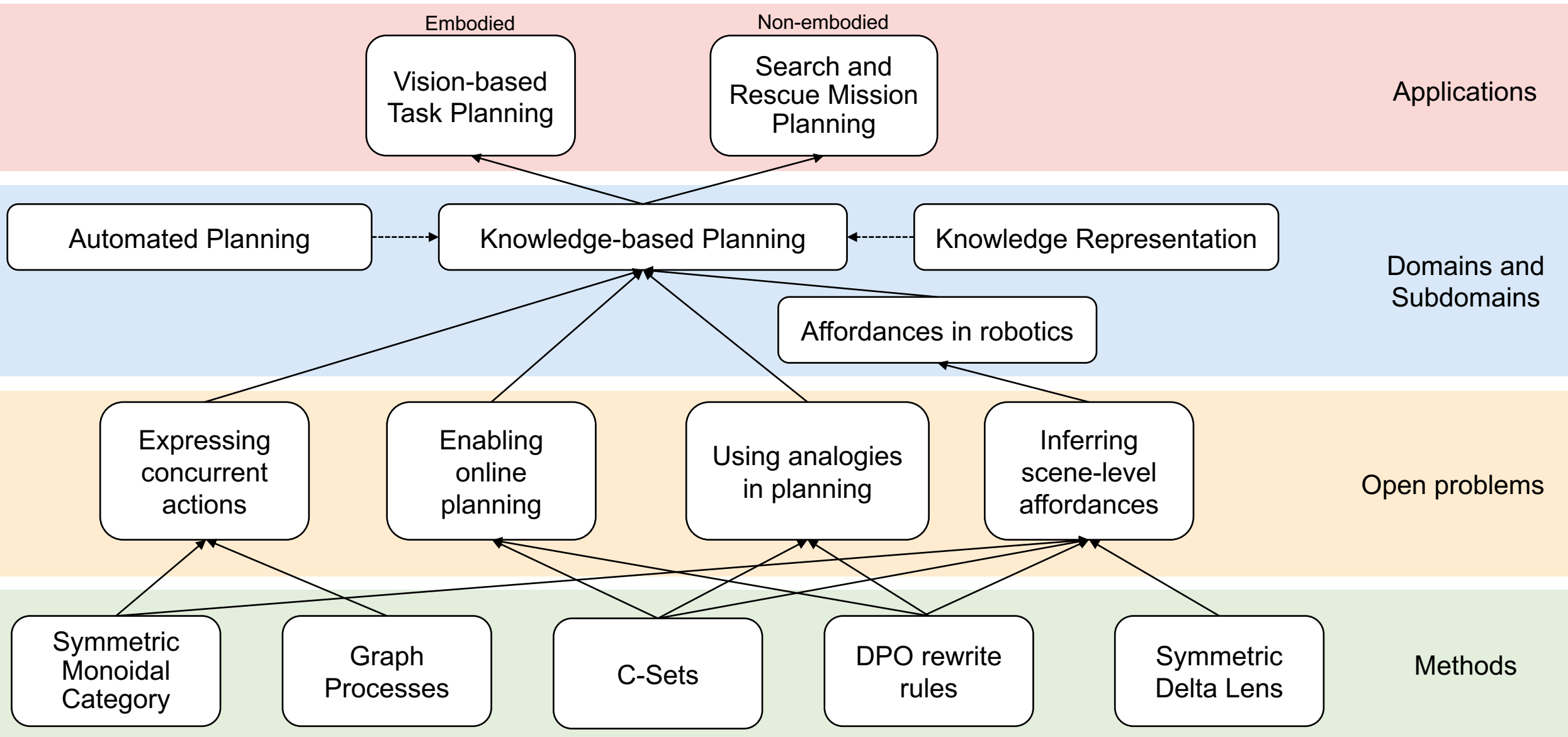
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AMS Joint Mathematics Meeting: Applied Category Theory Special Session
Boston, MA
January 2023

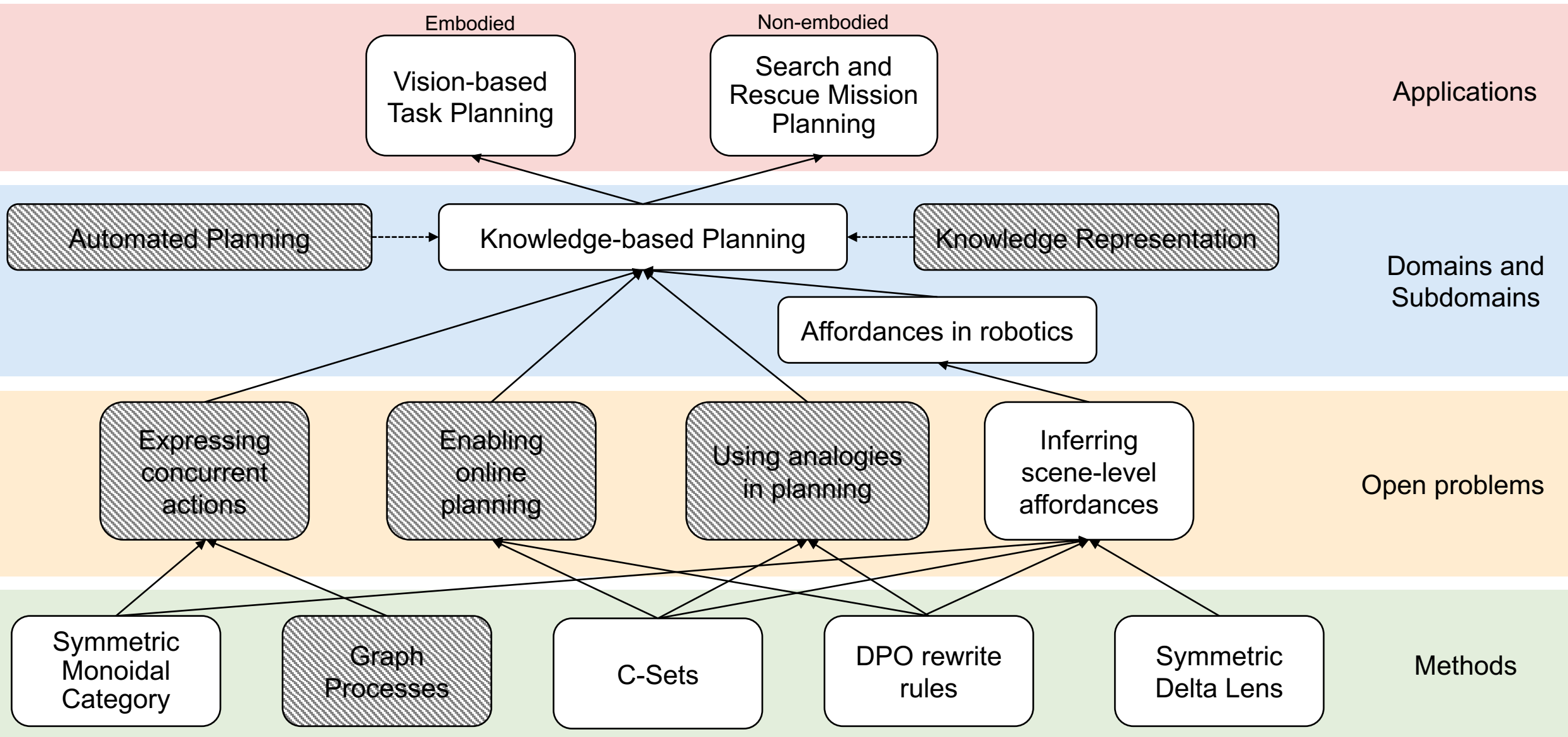
Contents

1. Motivating example
2. Using symmetric delta lenses for the affordance relation
 - What are the structures?
 - What kind of queries can we answer?
3. Ongoing work
 - Developing a categorical database using AlgebraicJulia
 - Future work

Technical Roadmap



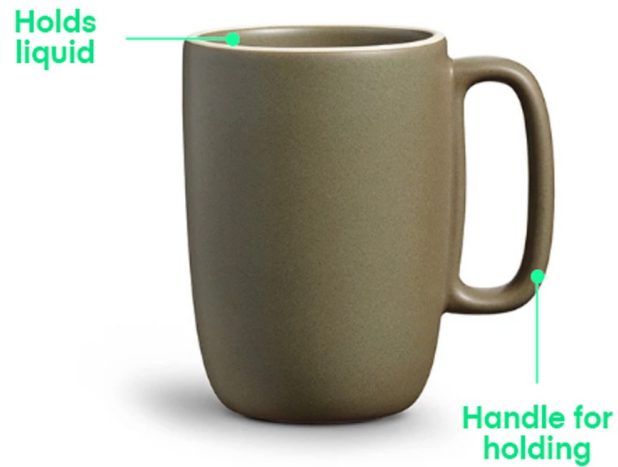
Technical Roadmap



Scene-level affordances in robotics

Mug vs. Mug with coffee on desk

<https://bootcamp.uxdesign.cc/what-every-game-ux-designer-should-know-about-human-psychology-9f0a325e919e>



Object-level



Scene-level

High-level actions often depend on **scene-level arrangements**, as opposed to, object-level features. There is little to no work done towards inferring scene-level affordances. [Lüddecke 2016]

Motivating Example: Kitchen World

Actions

```
(:action open-object
 :parameters (?obj - Object)
 :precond (not (openness ?obj))
 :effect (openness ?obj))

(:action close-object
 :parameters (?obj - Object)
 :precond (openness ?obj)
 :effect (not (openness ?obj)))

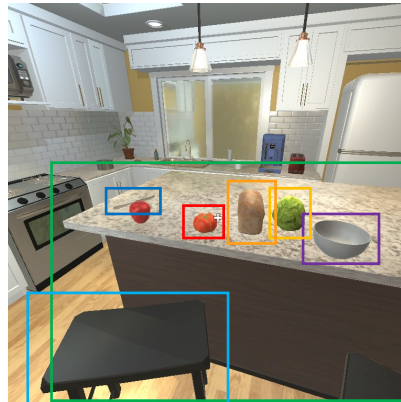
(:action cook-object
 :parameters (?obj - Object)
 :precond (not (cooked ?obj))
 :effect (cooked ?obj))

(:action slice-object
 :parameters (?obj - Object)
 :precond (not (sliced ?obj))
 :effect (sliced ?obj))

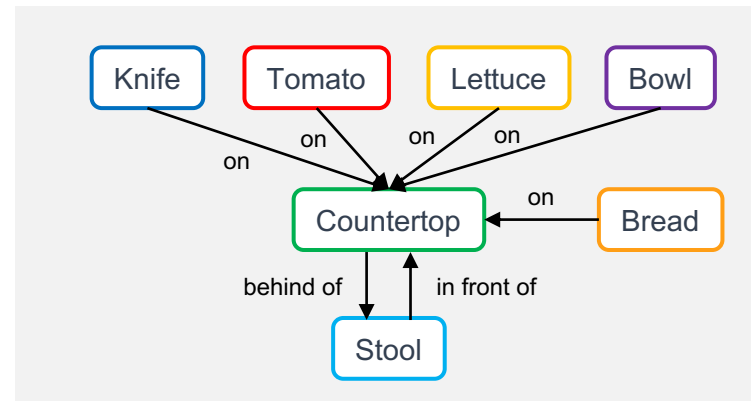
(:action pick-up-object
 :parameters (?target-obj - Object
 ?support-obj - Object ?agent - Agent)
 :precond (and (not (has ?agent ?target-obj))
 (on ?target-obj ?support-obj))
 :effect (and (has ?agent ?target-obj)
 (not (on ?target-obj ?support-obj))))

(:action put-object
 :parameters (?target-obj - Object
 ?support-obj - Object ?agent - Agent)
 :precond (and (has ?agent ?target-obj)
 (not (on ?target-obj ?support-obj)))
 :effect (and (on ?target-obj ?support-obj)
 (not (has ?agent ?target-obj))))
```

Scenes



Scene Graph



Afforded Actions

```
(:action pick-up-object
 :parameters (?target-obj - Object
 ?support-obj - Object ?agent - Agent)
 :precond (and (not (has ?agent ?target-obj))
 (on ?target-obj ?support-obj))
 :effect (and (has ?agent ?target-obj)
 (not (on ?target-obj ?support-obj))))
```

Motivating Example: Kitchen World

Actions

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(:action open-object
 :parameters (?obj - Object)
 :precond (not (openness ?obj))
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 :precond (openness ?obj)
 :effect (not (openness ?obj)))

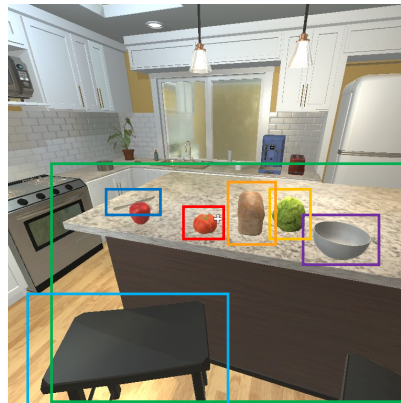
(:action cook-object
 :parameters (?obj - Object)
 :precond (not (cooked ?obj))
 :effect (cooked ?obj))

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 :effect (sliced ?obj))

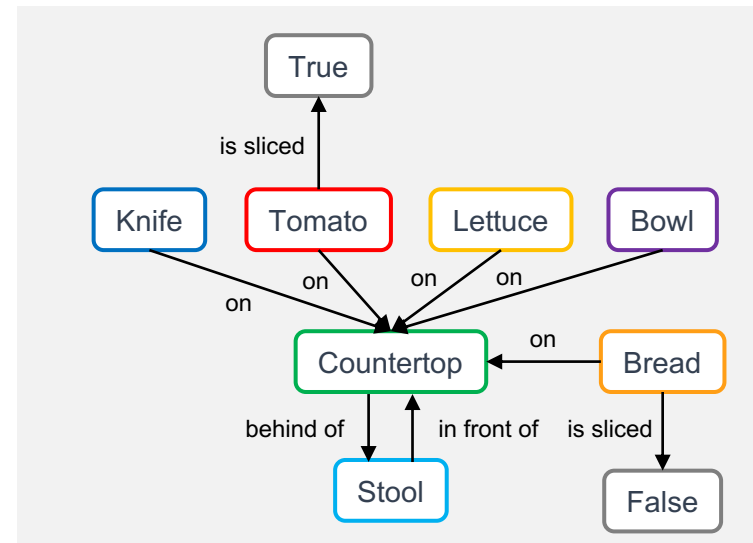
(:action pick-up-object
 :parameters (?target-obj - Object
 ?support-obj - Object ?agent - Agent)
 :precond (and (not (has ?agent ?target-obj))
 (on ?target-obj ?support-obj))
 :effect (and (has ?agent ?target-obj)
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(:action put-object
 :parameters (?target-obj - Object
 ?support-obj - Object ?agent - Agent)
 :precond (and (has ?agent ?target-obj)
 (not (on ?target-obj ?support-obj)))
 :effect (and (on ?target-obj ?support-obj)
 (not (has ?agent ?target-obj))))
```

Scenes



Scene Graph



Afforded Actions

```
(:action slice-object
 :parameters (?obj - Object)
 :precond (not (sliced ?obj))
 :effect (sliced ?obj))

(:action pick-up-object
 :parameters (?target-obj - Object
 ?support-obj - Object ?agent - Agent)
 :precond (and (not (has ?agent ?target-obj))
 (on ?target-obj ?support-obj))
 :effect (and (has ?agent ?target-obj)
 (not (on ?target-obj ?support-obj))))
```

Motivating Example: Kitchen World

Actions

```
(:action open-object
:parameters (?obj - Object)
:precond (not (openness ?obj))
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(:action close-object
:parameters (?obj - Object)
:precond (openness ?obj)
:effect (not (openness ?obj)))

(:action cook-object
:parameters (?obj - Object)
:precond (not (cooked ?obj))
:effect (cooked ?obj))

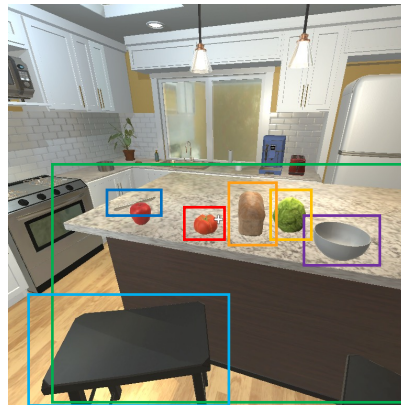
(:action slice-object
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:effect (sliced ?obj))

(:action pick-up-object
:parameters (?target-obj - Object
?support-obj - Object ?agent - Agent)
:precond (and (not (has ?agent ?target-obj))
(on ?target-obj ?support-obj))
:effect (and (has ?agent ?target-obj)
(not (on ?target-obj ?support-obj))))

(:action put-object
:parameters (?target-obj - Object
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:precond (and (has ?agent ?target-obj)
(not (on ?target-obj ?support-obj)))
:effect (and (on ?target-obj ?support-obj)
(not (has ?agent ?target-obj))))
```

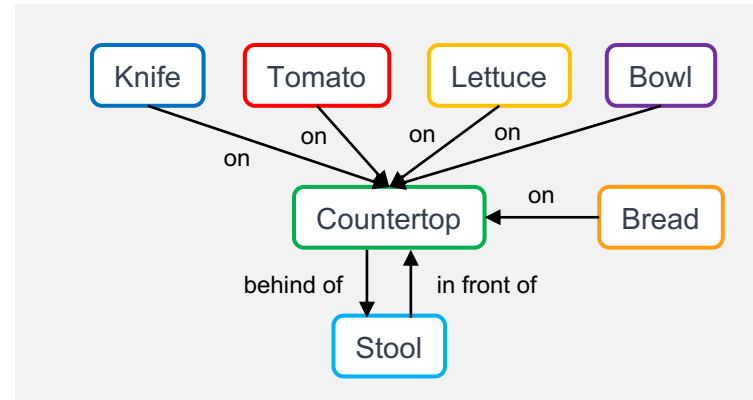
PDDL

Scenes

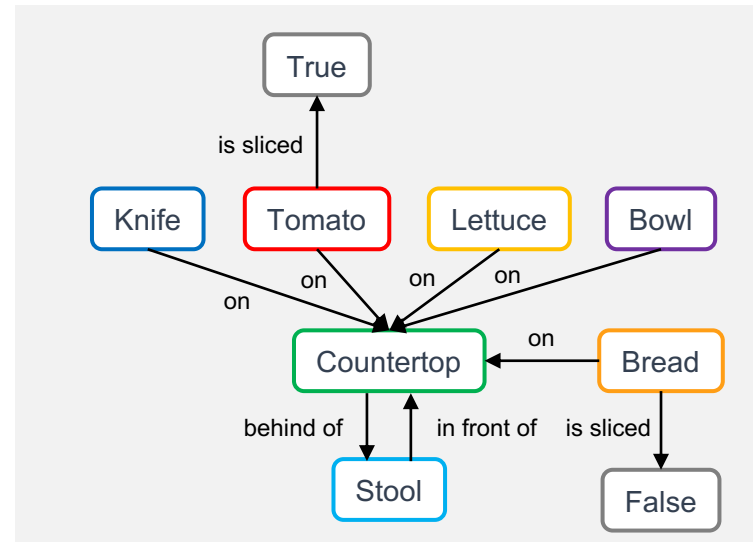


AI2THOR

Initial Scene Graph



Δ_G



Afforded Actions

```
(:action pick-up-object
:parameters (?target-obj - Object
?support-obj - Object ?agent - Agent)
:precond (and (not (has ?agent ?target-obj))
(on ?target-obj ?support-obj))
:effect (and (has ?agent ?target-obj)
(not (on ?target-obj ?support-obj))))
```

Δ_A

```
(:action slice-object
:parameters (?obj - Object)
:precond (not (sliced ?obj))
:effect (sliced ?obj))

(:action pick-up-object
:parameters (?target-obj - Object
?support-obj - Object ?agent - Agent)
:precond (and (not (has ?agent ?target-obj))
(on ?target-obj ?support-obj))
:effect (and (has ?agent ?target-obj)
(not (on ?target-obj ?support-obj))))
```


Using symmetric delta lenses for the affordance relation

Method

\mathbb{C} – Set and Attributed \mathbb{C} – Set

Def. A (finite) \mathbb{C} – Set is a functor from $\mathbb{C} \rightarrow \text{FinSet}$.

For computable examples, we assume finitely presented categories.

Def. A (finite) **attribute** \mathbb{C} – Set is a functor, F , from a finitely presented schema category, \mathbb{C} , to Set , where \mathbb{C} is partitioned using a map $S: \mathbb{C} \rightarrow \mathbf{2}$.

The preimage $S^{-1}(0)$ isolates the combinatorial structure.

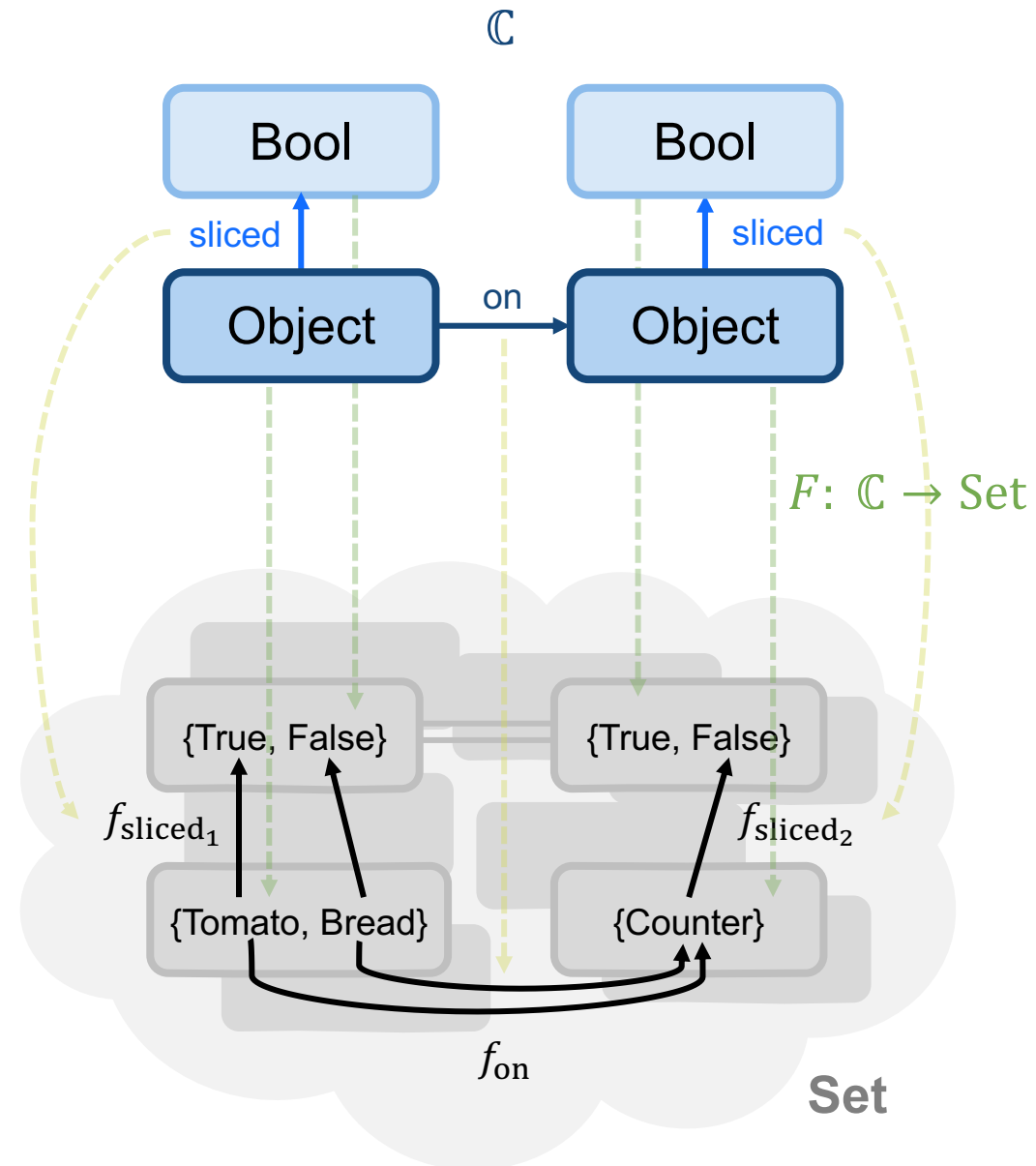
The preimage $S^{-1}(1)$ isolates the attribute structure.

The preimage $S^{-1}(0 \rightarrow 1)$ isolates the arrows between the combinatorial structure and the attribute structure.

Note: The Grothendieck construction, $\int F$, translates to RDF triples, e.g.

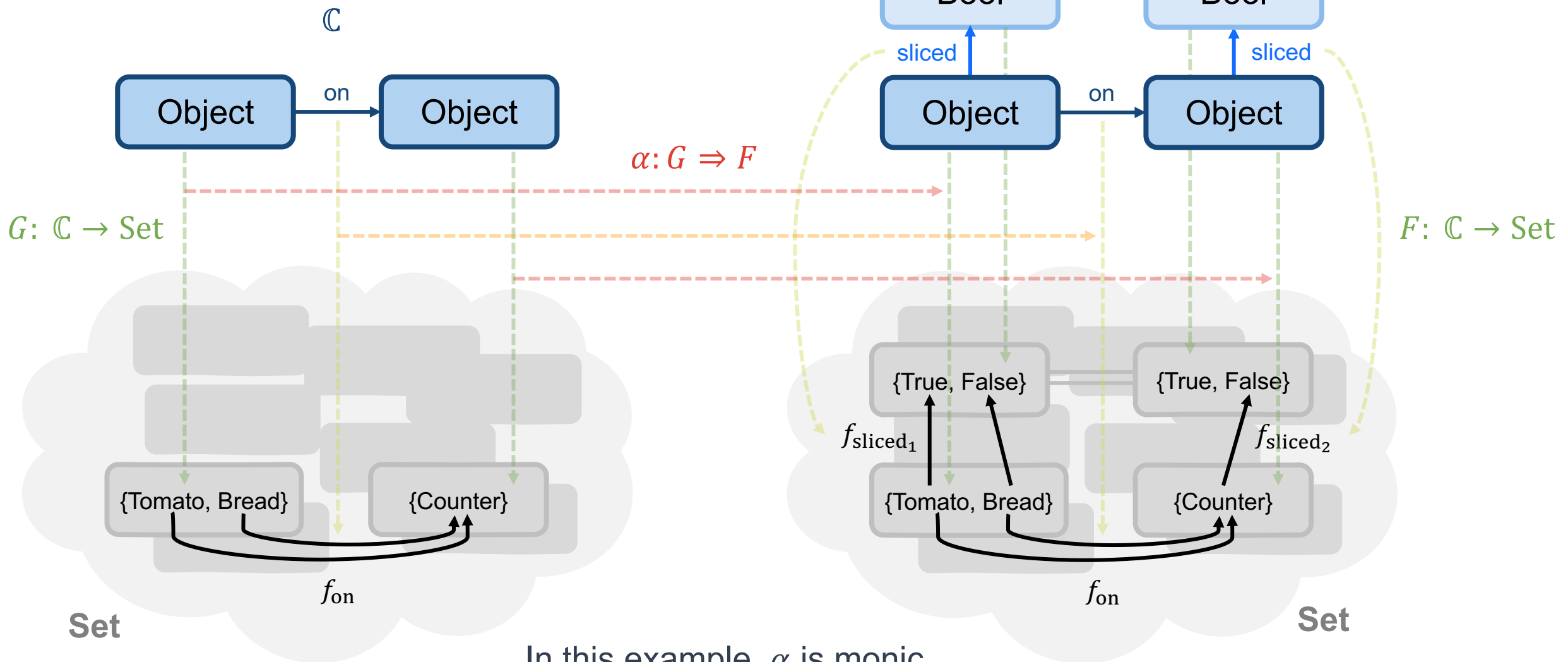
(Tomato :: Object, f_{on} :: on, Counter :: Object)

(Tomato :: Object, f_{sliced_1} :: sliced, True :: Bool)



Map between \mathbb{C} – Sets

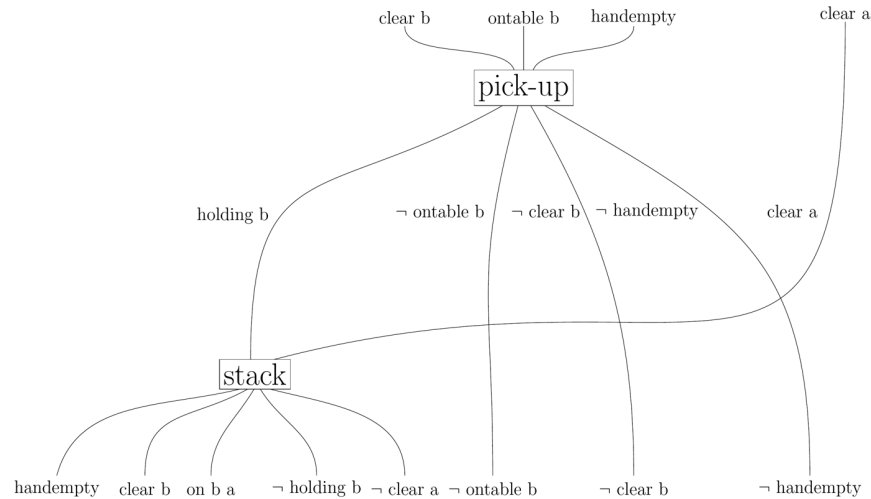
Is a natural transformation



In this example, α is monic.

Language of Planning Domains

Planning domains are a set of atomic action operators that can be composed to form a sequence of actions, or task plan.



Categorically, a STRIPS-based planning domain can be represented as a **symmetric monoidal category** where the generating objects are literals, the generating arrows are action operators, and the tensor product is conjunction. Positive and negated sentences are considered unique objects with no relation.

Aguinaldo A., Regli W. Encoding Compositionality in Classical Planning Solutions. IJCAI Workshop on Generalization in Planning 2021.

Def. A *planning domain*, P , consists of a set of action schemas with parameters (`parameters`), preconditions (`precond`), effects (`effect`).

Preconditions and effects in an action operator consist of a conjunction of fluents.

```
(:action pick-up-object
  :precond (and (not (has MyRobo Tomato)) (on Tomato Counter))
  :effect (and (has MyRobo Tomato) (not (on Tomato Counter))))
```

A set of action operators can be lifted to be universally quantified over all variables to form an action schema. Preconditions and effects in an action operator consist of a conjunction of literals.

```
(:action pick-up-object
  :parameters (?target-obj - Object ?support-obj - Object
    ?agent - Agent)
  :precond (and (not (has ?agent ?target-obj)) (on
    ?target-obj ?support-obj))
  :effect (and (has ?agent ?target-obj)
    (not (on ?target-obj ?support-obj))))
```

Symmetric monoidal categories

Def. A *symmetric monoidal category*, \mathbb{M} , is a category with the following additional properties:

- A unit object, $I \in \mathbb{M}$
- A tensor product, $\otimes: \mathbb{M} \times \mathbb{M} \rightarrow \mathbb{M}$
- An associative isomorphism,

$$\alpha_{X,Y,Z}: (X \otimes Y) \otimes Z \rightarrow X \otimes (Y \otimes Z)$$

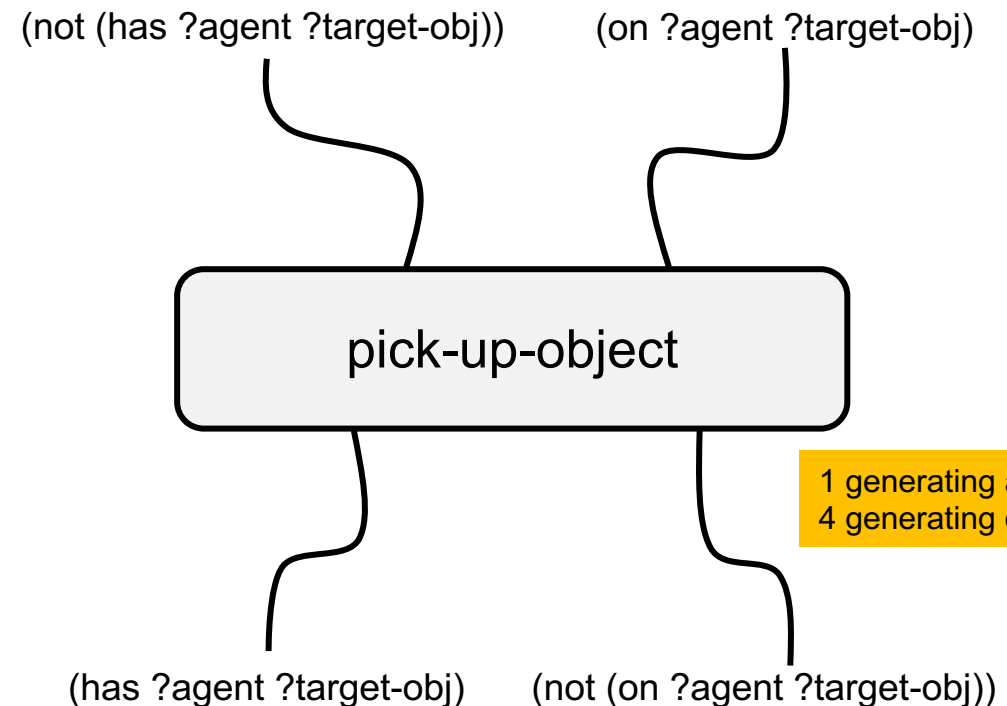
- Left and right unitor isomorphisms,

$$\rho_L: I \otimes X \rightarrow X \text{ and } \rho_R: X \otimes I \rightarrow X$$

- And a braiding isomorphism,

$$B_{X,Y}: X \otimes Y \rightarrow Y \otimes X$$

```
(:action pick-up-object
  :parameters (?target-obj - Object ?support-obj - Object
    ?agent - Agent)
  :precond (and (not (has ?agent ?target-obj)) (on
    ?target-obj ?support-obj))
  :effect (and (has ?agent ?target-obj)
    (not (on ?target-obj ?support-obj))))
```



Functor between symmetric monoidal categories

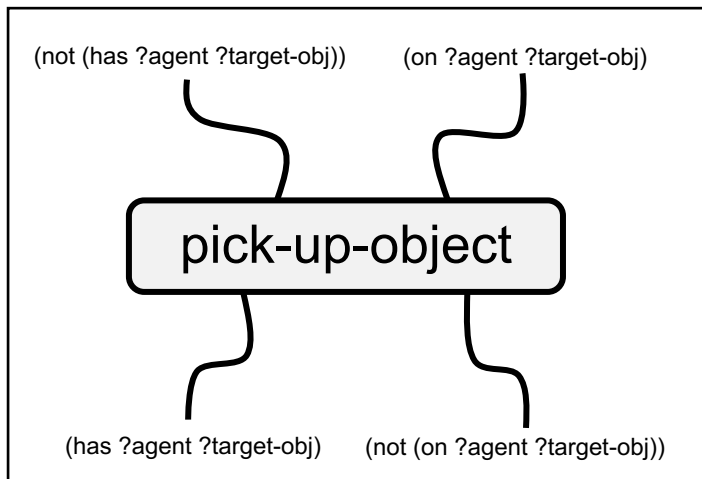
Symmetric Monoidal Category, \mathbb{M}

Objects:

- $X = (\text{not } (\text{has } ? \text{agent } ? \text{target} - \text{obj}))$ ----->
- $Y = (\text{on } ? \text{agent } ? \text{target} - \text{obj})$ ----->
- $Z = (\text{has } ? \text{agent } ? \text{target} - \text{obj})$ ----->
- $U = (\text{not } (\text{on } ? \text{agent } ? \text{target} - \text{obj}))$ ----->

Arrows:

- pick-up-object: $X \otimes Y \rightarrow Z \otimes U$ ----->



$H: \mathbb{M} \rightarrow \mathbb{M}'$

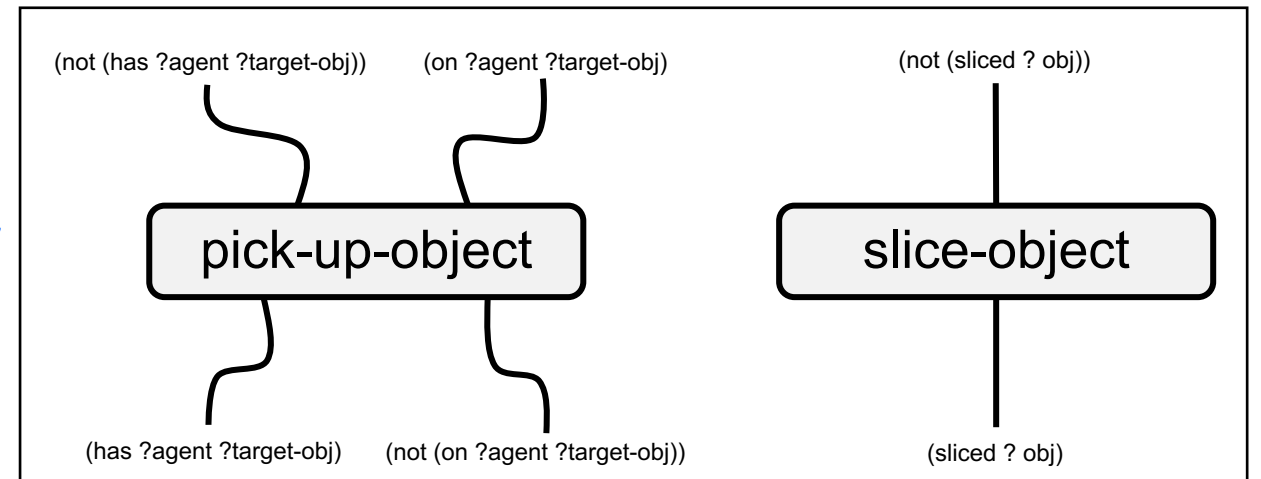
Symmetric Monoidal Category, \mathbb{M}'

Objects:

- $X = (\text{not } (\text{has } ? \text{agent } ? \text{target} - \text{obj}))$
- $Y = (\text{on } ? \text{agent } ? \text{target} - \text{obj})$
- $Z = (\text{has } ? \text{agent } ? \text{target} - \text{obj})$
- $U = (\text{not } (\text{on } ? \text{agent } ? \text{target} - \text{obj}))$
- $T = (\text{not } (\text{sliced } ? \text{obj}))$
- $R = (\text{sliced } ? \text{obj})$

Arrows:

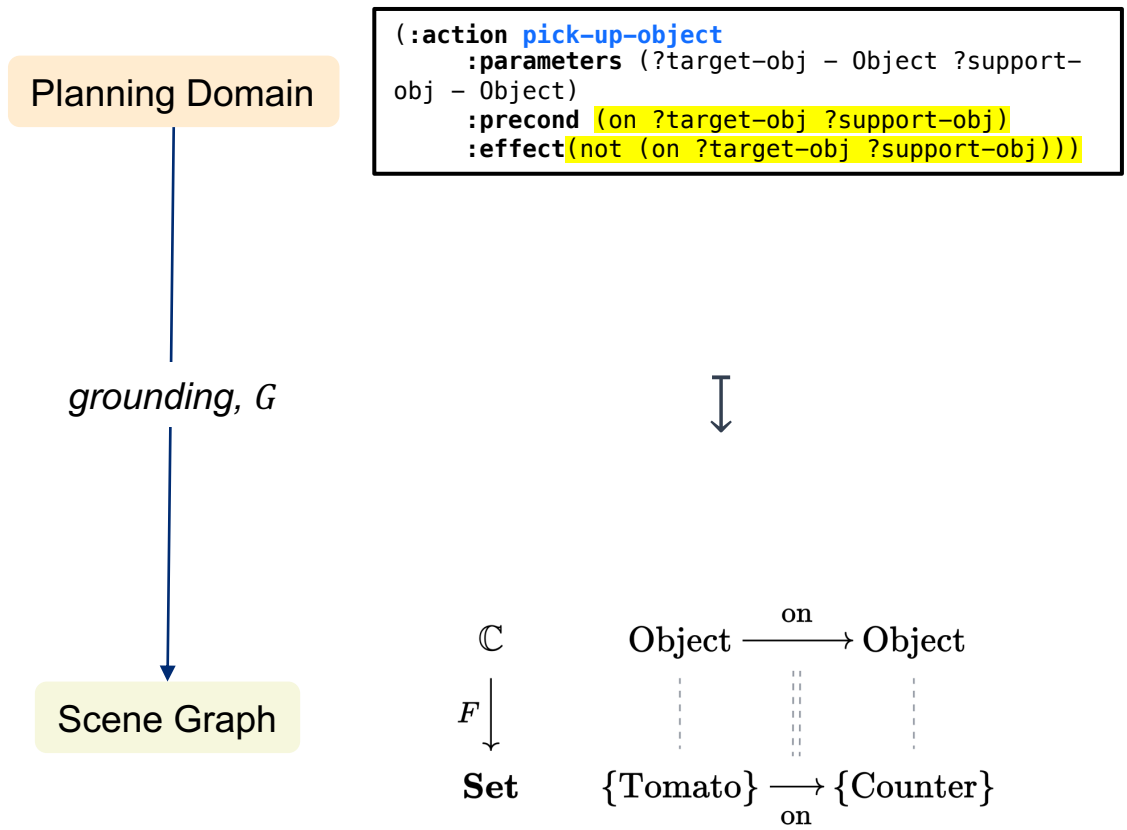
- pick-up-object: $X \otimes Y \rightarrow Z \otimes U$
- slice-object: $T \rightarrow R$



Symmetric monoidal functor, H , is a functor that preserves monoidal and braiding isomorphisms.

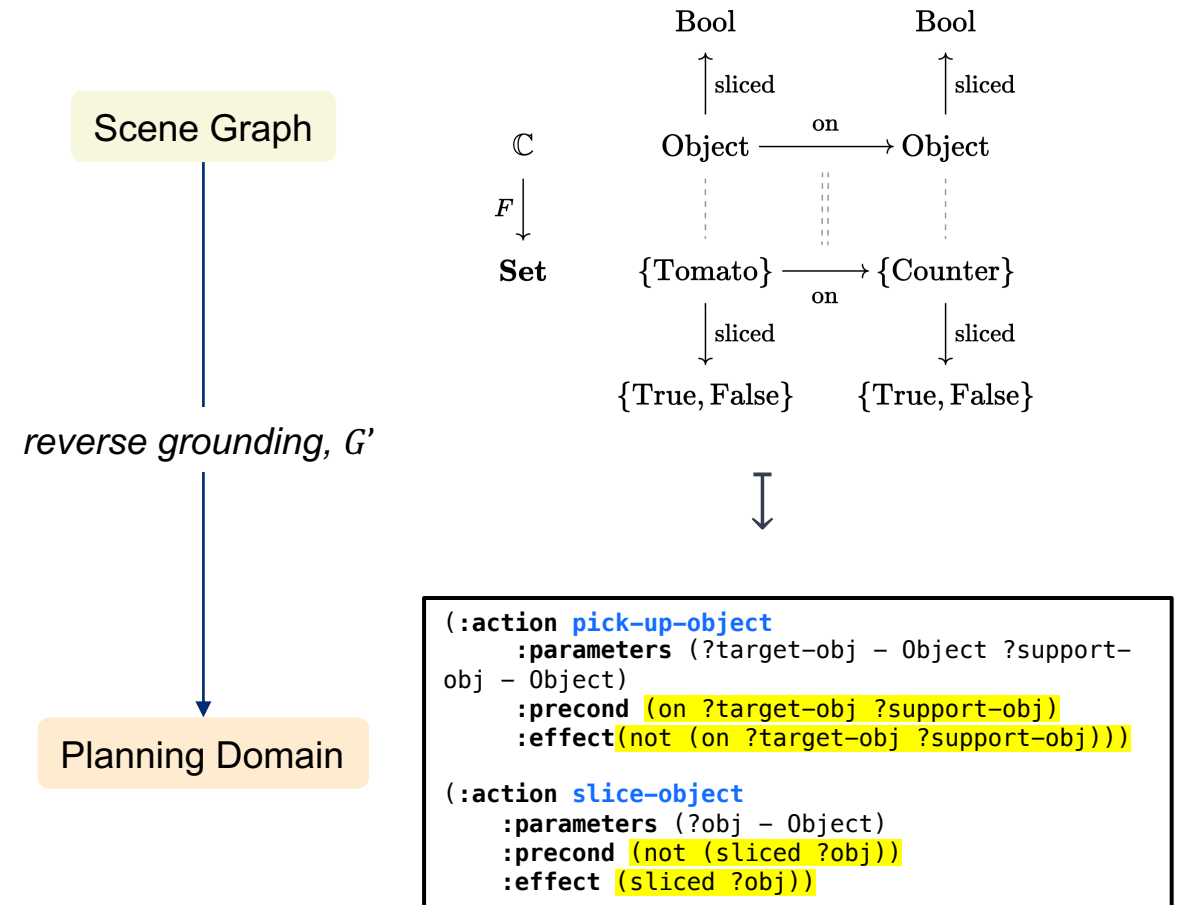
Affordance relation using functors: Object maps

Functor G

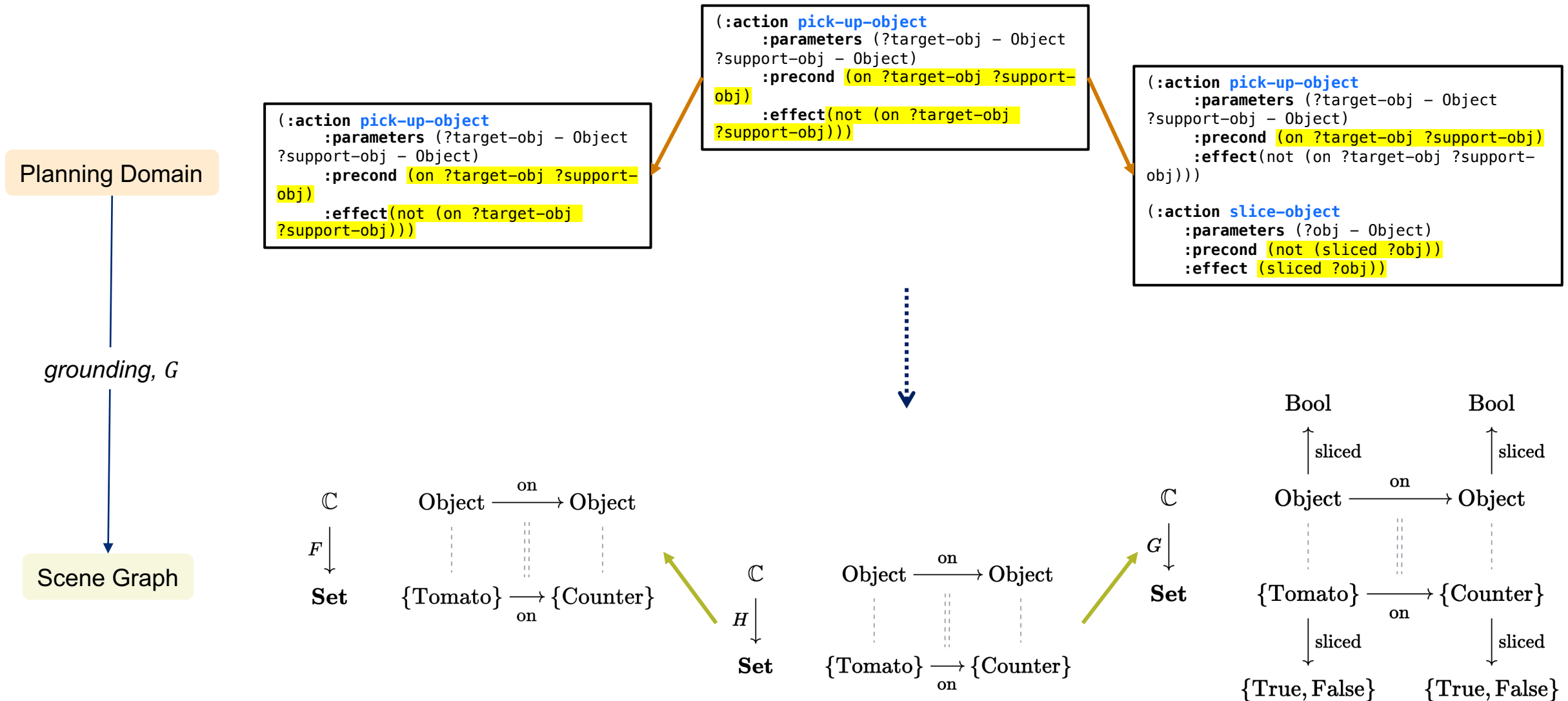


Showing only object maps

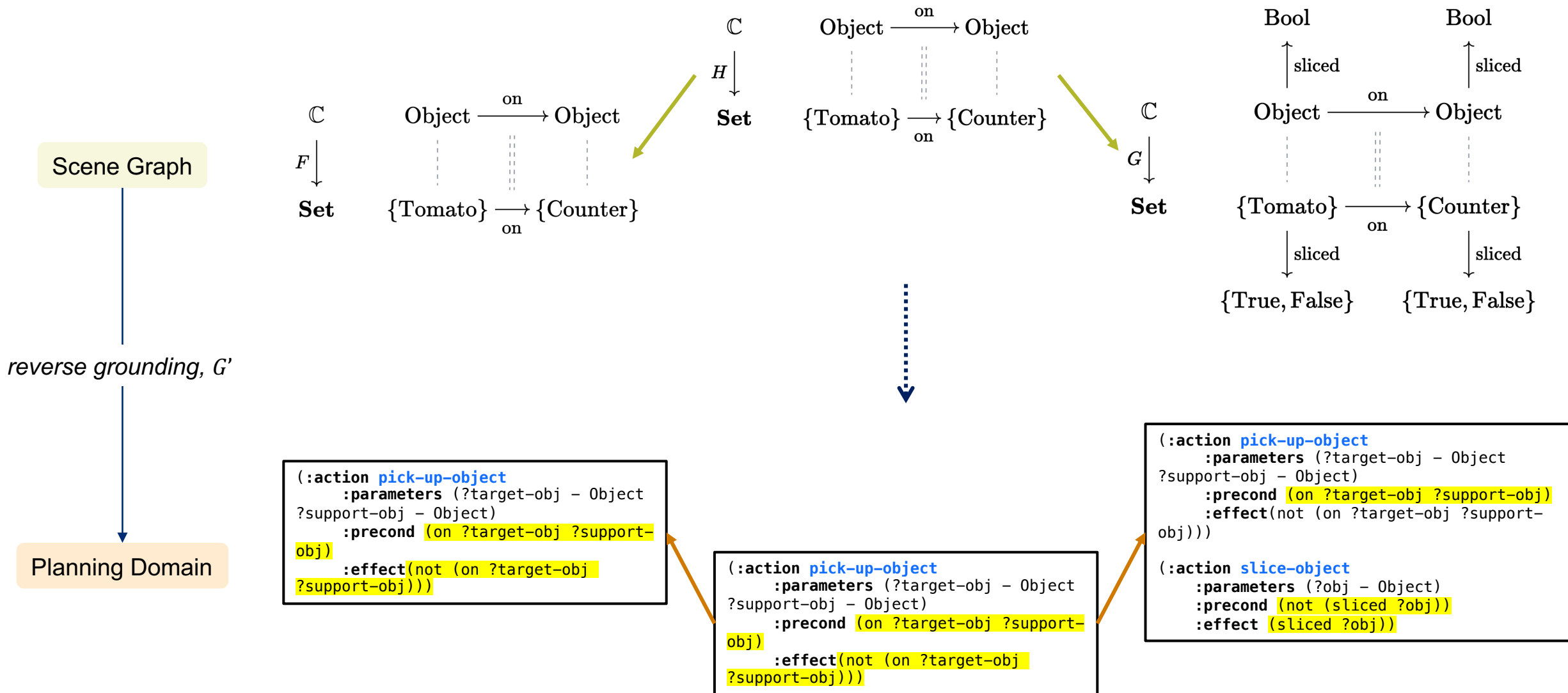
Functor G'



Affordance relation using functors: Arrow maps



Affordance relation using functors: Arrow maps



Inferring affordances using symmetric delta lens

Claim. Symmetric delta lens construct the affordance relation.

Def. Symmetric delta lenses

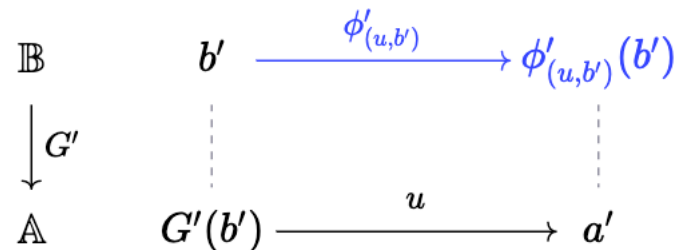
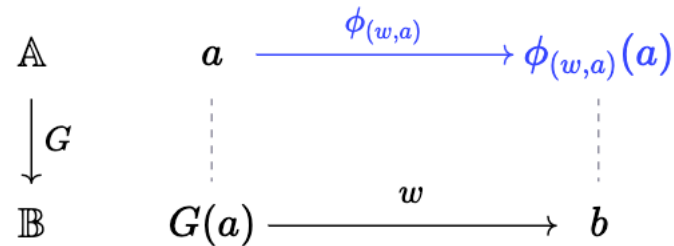
1. Delta lens $(G, \phi): \mathbb{A} \rightarrow \mathbb{B}$
2. Delta lens $(G', \phi'): \mathbb{B} \rightarrow \mathbb{A}$

Axioms

Lifting operations, ϕ, ϕ' , provides unique lifts. They preserve compositions and identities.

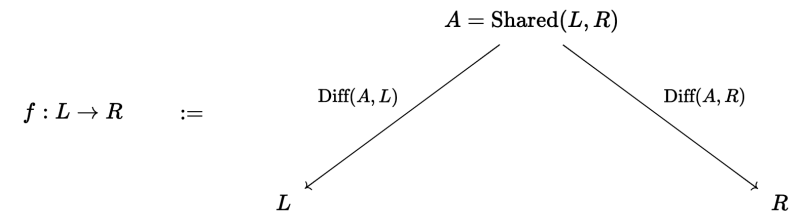
Functors, G, G' , are arbitrary functors.

Johnson 2016



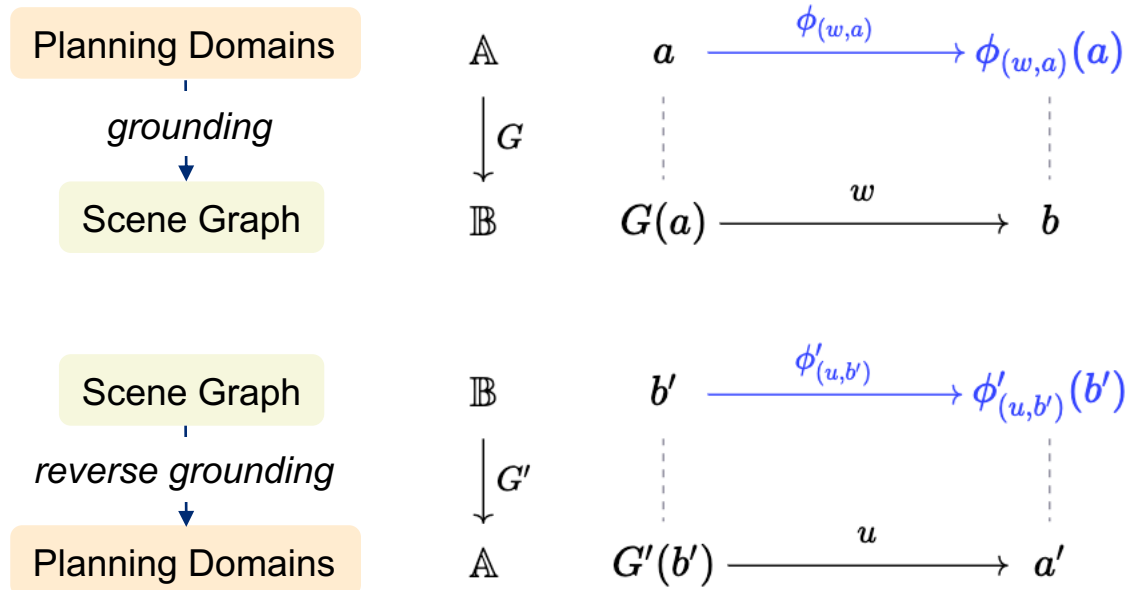
Within each category, (\mathbb{A}, \mathbb{B})

- Objects are models
- Arrows, f , are model updates (deltas)



$\mathbb{A} \sim$ category of planning domains
 $\mathbb{B} \sim$ category of scene graphs

What kind of queries can we answer?



Queries

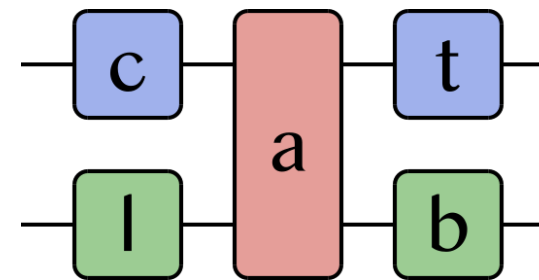
- i. What is $G(a)$? “What scene graph is afforded by this planning domain?”
- ii. What is $\phi_{(w,a)}$? “Given a change in the scene graph, what changes in the afforded planning domains?”
- iii. What is $G'(b')$? “What planning domain is afforded by this scene graph?”
- iv. What is $\phi'_{(u,b')}$? “Given a change in the afforded planning domains, what changes are necessary in the scene graph?”

Ongoing Work

Operationalization and evaluation

Computational categories in development

```
1 using Catlab, Catlab.Theories
2 using AlgebraicPlanning
3
4 # Schema
5 #####
6
7 # Base schema
8 #-----
9
10 @present SpecKitchen(FreeMCategory) begin
11   Entity::Ob
12
13   Food::Ob
14   food_in_on::Hom(Food, Entity)
15   food_is_entity::Hom(Food, Entity)
16   ::Tight(food_is_entity)
17
18   Kitchenware::Ob
19   ware_in_on::Hom(Kitchenware, Entity)
20   ware_is_entity::Hom(Kitchenware, Entity)
21   ::Tight(ware_is_entity)
22 end
23
24 function add_food!(pres::Presentation, name::Symbol)
25 | add_entity!(pres, name, type=:Food)
26 end
27 function add_kitchenware!(pres::Presentation, name::Symbol)
28 | add_entity!(pres, name, type=:Kitchenware, is_a=:is_ware)
29 end
30
31 function add_entity!(pres::Presentation{MCategory}, name::Symbol;
32 | | | | | | | | type::Symbol=:Entity, is_a::Union{Symbol,Nothing}=nothing)
33 | isnothing(is_a) && (is_a = Symbol("is_", snakecase(type)))
34 | ob = add_generator!(pres, Ob(FreeMCategory, name))
35 | is_a_name = Symbol(snakecase(name), "_", is_a)
36 | is_a_hom = add_generator!(pres, Hom(is_a_name, ob, pres[type]))
37 | add_generator!(pres, Tight(nothing, is_a_hom))
38 end
39
```



<https://github.com/AlgebraicJulia/Catlab.jl>

Features

- C-sets (copresheaves)
- Symmetric monoidal categories
- Categorical database migration
- RDF to C-set serialization
- PDDL to SMC serialization
- Lenses

In collaboration with Evan Patterson, James Fairbanks, Owen Lynch, Kris Brown, Sophie Libkind

A. Aguinaldo. Using categorical logic for AI planning. 2022. Blogpost: <https://www.algebraicjulia.org/blog/post/2022/09/ai-planning-cset/>

Thanks for listening!

Please feel free to reach out with questions, suggestions, or related projects.

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